Accurate "superluminal" transmission via entanglement, superoscillations and quasi-Dirac distributions

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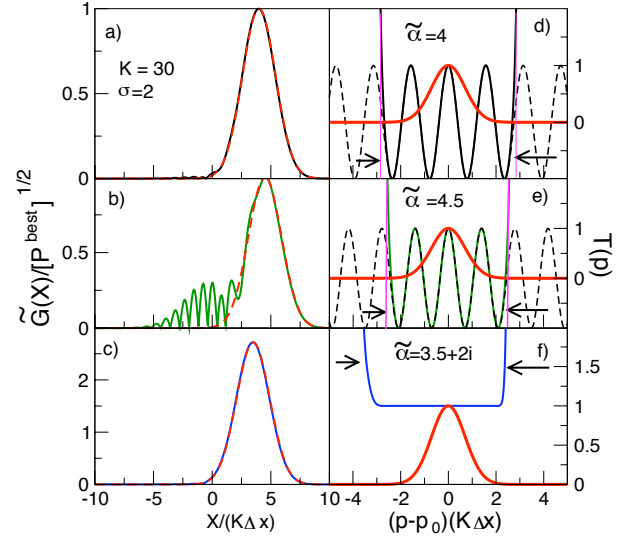
We analyse a system in which, due to entanglement between the spin and spatial degrees of freedom, the reduced transmitted state has the shape of the freely propagating pulse translated in the complex co-ordinate plane. In the case an apparently "superluminal" advancement of the pulse the delay amplitude distribution is found to be a peculiar approximation to the Dirac δ-function, and the transmission coefficient exhibits a well-defined super-oscillatory window. Analogies with potential tunnelling and the Wheeler's delayed choice experiment are highlighted.

Introduction. Quantum speed up effect known since early thirties of last century allows the maximum of a wavepacket, transmitted across a classically forbidden region, arrive at a detector earlier than that of a freely propagating one. It has been predicted and observed for various systems such as potential barriers, semi-transparent mirrors, refraction of light, microwaves in undersized wave guides and fast-light materials [1], [2] A transmission appears "superluminal" if one uses the advancement of the transmitted peak to predict the time spent by the particle, photon or electromagnetic pulse inside a barrier, scatterer or a waveguide. It is generally understood that such a prediction is incorrect since the initial pulse, usually greatly reduced, undergoes in the process a severe reshaping.

Superoscillatory functions which, in a limited region, oscillate with a frequency outside the support of their Fourier spectrum were introduced in [3] and extensively studied in [4]. These functions have been recently applied for improving optical resolution beyond the diffraction limit [5]. Authors of Ref. [6] have established a connection between superoscillatory behaviour and anomalous tunnelling times (weak values) which occur in "superluminal" signal transmission. The analogy between apparent "superluminality" and weak measurements was further explored in Ref. [7], and additional work on the weak values and superoscillations can be found in [8].

The purpose of this paper is to propose and analyse a system in which a similar speed up effect occurs due to entanglement between the spin and spatial degrees of freedom in a magnetic field so that the transmitted state is essentially a reduced copy of the freely propagating one, with an additional coordinate shift. We will show that in the case of a significant advancement the delay amplitude distribution [7] (DAD) is a wildly oscillatory function with a finite support Ω which approximates a Dirac δ-function with support outside Ω. In the momentum space, the transmission coefficient exhibits a superoscillatory behaviour within a well defined band.

We will also demonstrate that manipulations of the particle's spin states allows to choose between the regimes where the particle spend a known amount of time in the magnetic field and where, in the spirit of wave-particle duality, such a duration cannot be defined.

Quantum speed up effect via entanglement. We consider a particle of a unit mass $\mu = 1$ equipped with a $2K + 1$-component magnetic moment (spin). The particle is described by a wave packet with a mean momentum p_0, which propagates without spreading in the time interval of interest. Thus, its freely propagating wave function is given by

$$\langle x|\Psi(t)\rangle = \exp(ip_0 x - ip_0^2 t/2)G(x - p_0 t)|a\rangle \quad (1)$$

where G is the envelope and $|a\rangle \equiv \sum_{m=-K}^{K} a_m |m\rangle / \sqrt{N(a)}$

$$N(a) \equiv \sum_{m=-K}^{K} |a_m|^2 \quad (2)$$

is the initial state of the spin written here in terms of its components, m, along the z-axis. Next we make the particle pass through a constant magnetic field in the z-direction created in the region $0 < x < d$ so that the m-th component of the spin wave function encounters there an additional rectangular potential of the magnitude $m\omega_L$ (ω_L is the Larmor frequency), a well if $m < 0$ and a barrier for $m > 0$. If the particle is sufficiently fast, $p_0^2/2 >> K\omega_L$, the reflection off the field's edges can be neglected, and upon traversing the field the m-th component of the wave function will be advanced or lagging behind the freely propagating pulse by $m\Delta x$, $\Delta x \equiv \omega_L d/p_0^2$. On exit from the magnetic field the wave function contains a superposition of shifted wave packet's envelopes, with shifts ranging from $-K\Delta x$ to $K\Delta x$,

$$\langle x|\Psi(t)\rangle = \exp(ip_0 x - ip_0^2 t/2) \sum_{m=-K}^{K} a_m \times \quad (3)$$

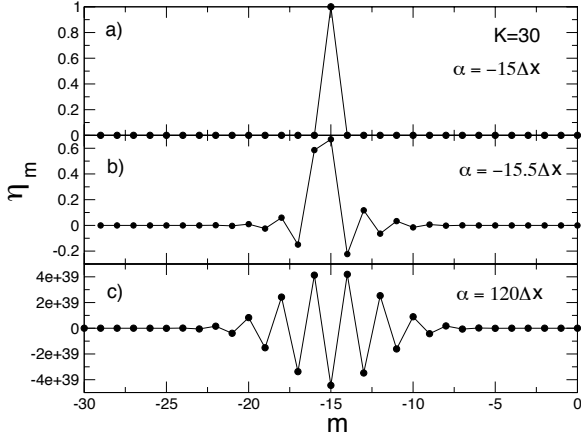$$\exp(-im\omega_L d/p_0)G(x - p_0 t - m\Delta x)|m\rangle / \sqrt{N(a)}.$$

$\alpha = \alpha_1 + i\alpha_2$, in which case one with a complex frequency, we have $T(p) \approx \exp(-ip\alpha_1 + p\alpha_2)$, and find the free envelope in Eq.(6) analytically continued into the complex coordinate plane. In general, in order to avoid distortion we need the momentum distribution of the initial wavepacket $A(p)$ to fit into the superoscillatory band of the transmission coefficient, beyond which $T(p)$ exhibits rapid (polynomial) growth similar to that reported in Refs. [3],[6] (Fig.3). For a Gaussian envelope (12) this requires (c.f. Eq.(16)) $2/\sigma << K/(e|\alpha|)$ which puts a restriction on the minimal width of the wave packet which can be translated without distortion.

Conclusions and discussion. In summary, we have proposed a system for which the transmitted state is a reduced copy of the freely propagating one, translated along an arbitrary direction into the complex coordinate plane. The shift is achieved by splitting the initial pulse into several copies delayed relative to free propagation. The system can be described in terms of a transmission amplitude $T(p)$ or, alternatively, by its Fourier transform, the delay amplitude distribution (DAD) $\eta(x)$. The DAD contains information about the spectrum of virtual delays experienced by the particle in the magnetic field and, therefore, about causality. For a real translation the DAD is a finite order approximation to a Dirac δ-function and, for a complex valued shift, an analytic continuation of such an approximation. Accordingly, $T(p)$ exhibits a superoscillatory band whose width limits the size of the shift and the width of the initial pulse. The bandwidth depends on the number of first moments of the DAD (momentum derivatives of $T(p)$ at $p = 0$) which satisfy $\bar{x^n} \approx \alpha^n$. A similar analysis can be applied to any system described by a transmission amplitude which is analytic in the complex momentum plane, e.g., tunnelling across a potential barrier [7], where the shift is determined by the barrier width and the DAD, like that in Eq. (4) contains only delayed contributions [7].

It is worth noting again that certain contradictions in the analysis of the so-called tunnelling time problem [1] arise in an attempt to deduce the time τ a particle spends in the scatterer from its delay $\Delta T = -\alpha/p_0$ in arriving at a remote detector. This suggests that the time spent in the scatterer is $\tau = d/p_0 - \alpha/p_0$ and the notion of apparent "superluminality" arises if τ is made to be less than d/c. Our model readily demonstrates that there is no simple relation between ΔT and τ. Indeed, between the field and the polariser the probability density corresponds, as it should, to K real delays, $P(x) = \sum_{m=-K}^{0} |a_m|^2 |G(x - p_0t - m\Delta x)|^2$. The advanced shape (6) is formed upon passing a (possibly remote) polariser, by an interference mechanism analogous to that responsible for anomalous values obtained in weak measurements [14]. There is also a similarity to the Wheeler's delayed choice experiment [15]. Just like Wheeler's photon may behave as a particle or as a wave, depending on which observation is made after the event,

the particle spends in the magnetic field a well defined duration $T = (d - m\Delta x)/p_0$ provided the polariser is set to select one of the states $|m\rangle$. If, on the other hand, the post-selection is performed in a linear combination of more than one of such states, this duration cannot be defined and the experiment probes the wave nature of the transmitted particle.

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[10] Consider the determinant $\Delta_m(\alpha)$ of the matrix A in which the m-th column is replaced by $(1, \alpha, \alpha^2, ...\alpha^K)$. $\Delta_m(\alpha)$, which is a polynomial of order K in α, must vanish for any $\alpha = -n\Delta x$, $n \neq m$, and, therefore, can be written as $\Delta_m(\alpha) = \prod_{n \neq m}(\alpha + n\Delta x) \times const$. Also, we have $det A = \Delta_j(-j\Delta x)$, for any $0 \leq j \leq K$, and the ratio $\Delta_m(\alpha)/det A$ yields Eq.(9).

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